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THE FORMATION OF NEUTRON STARS AND THEIR SURFACE PROPERTIES

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Abstract

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In this paper we will study the structure and evolution of pre-supernovae stars. Two simple models are studied. Both models indicate that, during the final stage a degenerate core developes in massive stars (independent of mass). When the mass of a degenerate core exceeds the Chandrasekhar mass limit, collapse will take place. This is a third instability which may cause a star to collapse. The other two instabilities have been discussed previously(ref.(3) and (4)).

During the collapse phase it is inevitable that neutron stars will be formed. Neutrino emission processes rapidly dissipate gravitational energy released during collapse. There is no chance for a collapsed star to restore to the density for a white dwarf.

It is not known if the collapsed core will cut itself off from the rest of the world gravitationally (and hence become invisible).

If neutron stars are visible, they will radiate strongly in the 3-10 Å x-ray band, but weakly in the visible region. A theory of the atmosphere of neutron stars is developed here. It is found that a neutron star will radiate strongly in the 3-10 Å x-ray band for around 1000 years after it is formed. Afterwards it dims out rapidly. It is proposed that a single measurement of the red shift of neutron star spectra will yield the distance of neutron stars to us. Other experimental possibilities are discussed.

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I. Introduction

About 30 years ago Chandrasekhar[1] proved that at zero temperature there is no equilibrium configuration for a star with a mass greater than $5.66/\mu_e^2 \odot$. (μ_e is the ratio of mass number to atomic number; $\odot$ = solar mass $= 2 \times 10^{33}$ g). For a star made of elements other than hydrogen, the mass limit is around $1.4\odot$. Oppenheimer and Volkoff[2] extended Chandrasekhar's theory to the relativistic region. They found that, for a perfect Fermi gas, the relativistic mass limit is only $0.76\odot$.

Massive stars exist. In order to avoid the difficulty of the mass limit dilemma in stellar evolution discussions, it has been generally assumed that a massive star can always get rid of its mass before it comes to the end of stellar evolution. It is easy to see just from energetic grounds how erroneous this idea is (Section III.1).

In this paper we discuss how a star reaches a dynamically unstable stage, the energetics during stellar collapse, and the observations of remnants of collapsed stars (neutron stars) if they can be observed at all. It is suggested that neutron stars may be visible in the $3\mathring{A}$ - $10\mathring{A}$ X-ray band for around a few thousand years[3].

II. Evolution of Pre-Supernova Stars

(i) Equations of Evolution

One possible mechanism (iron-helium phase change) for supernova collapse has been discussed extensively by Fowler and Hoyle elsewhere[4]. In this paper we consider an alternative approach which does not depend upon certain simple assumptions (such as Eq. (21) of this paper) which Hoyle and Fowler made in their paper.

When a star is contracting rapidly, one must include the acceleration in the equation of hydrostatic equilibrium. As long as the gravitational energy is small compared with the rest energy of the star, we can use the non-relativistic approximation. Assuming spherical symmetry, the equation for hydrostatic equilibrium is

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} - \rho\dot{v} \tag{1}$$

where P is the total pressure, ρ the density, G the gravitational constant, m the mass contained in a sphere of symmetry of radius r. v is the radial velocity at r, and $\dot{v}$ is given by

$$\dot{v} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} \tag{2}$$

It is more convenient to use m as one of the two independent variables, since m is independent of t. Then Eq. (1) becomes

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} - \frac{\dot{v}}{4\pi r^2} \tag{3}$$

From the definition of m we have

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \tag{4}$$

Also, we have the equation of continuity

$$\dot{\rho} + \rho \text{ div } v = \dot{\rho} + \rho \frac{\partial v}{\partial r} = 0 \tag{5}$$

Before a star reaches the iron-helium stage, contraction is induced by neutrino process which dissipates stellar energy. Let the rate of energy dissipation per unit mass be $\epsilon_d(\rho,T)$ (in our notation ϵ_d has a positive sign). The energy balance equation is

$$-T \frac{dS}{dt} = \epsilon_d(\rho,T) \tag{6}$$

where S is the entropy per unit mass. We have neglected ordinary radiative transfer processes. For the annihilation process[5] we have

$$\epsilon_d(\rho,T) = \frac{4\text{x}10^{15}}{\rho} \left(\frac{T}{10^9}\right) \text{ ergs/g-sec} \tag{7}$$

2. Approximate Solutions[6]

Let W be the gravitational energy of the star and E its total thermodynamic energy; then the total energy of the star E_T is given by

$$E_T = W + E \tag{8}$$

The virial theorem gives

$$-W = (1 + \eta)E \tag{9}$$

where η is a slowly varying constant depending on the gross thermodynamic properties of a star. For all cases, $1 > \eta > 0$. At $T \sim 6\text{x}10^9\ ^oK$, $\eta \approx 0.1$.

Approximately, one can write

$$W = -G\int \frac{m\,dm}{r} \approx -\frac{GM^2}{R} \tag{10}$$

where R is the radius of the star, and also

$$E = \int \epsilon\, dm \approx M \cdot R_g T \tag{11}$$

where ϵ, the thermodynamic energy per unit mass, is approximated by the expression for a perfect gas:

$$\epsilon = R_g T \tag{12}$$

and R_g is the gas constant ($R_g = 8.2 \times 10^7$ in c.g.s. units). From Eqs. (10) and (11), the following relations are obtained:

$$T = \frac{GM}{R_g(1+\eta)}\,\frac{1}{R} \tag{13}$$

$$W = -\eta E_T = -\eta M R_g T = -\frac{\eta G}{1+\eta}\,\frac{M^2}{R} \tag{14}$$

Since gravitational contraction is the only energy source available at $T \gtrsim 10^9\ ^oK$(7), above this temperature the energy balance equation becomes

$$-\frac{dW}{dt} = \int \epsilon_d\, dm \approx M\epsilon_d(\rho, T) \tag{15}$$

From Eq. (13) we can also obtain a dynamical relation between T and ρ. Since $\rho = \frac{4}{3}\pi R^3$, the relation is

$$\rho = \frac{R_g^3}{4G^3M^2}\,T^3 \equiv AT^3 \tag{16}$$

From Eqs. (14) - (16), we obtain the following relations:

$$-\frac{dR}{dt} = \frac{1+\eta}{\eta GM}\,\epsilon_o\,\frac{1}{A}\left[\frac{GM}{R_g(1+\eta)}\right]^6 \frac{1}{R^4} = \frac{C}{R^4} \tag{17}$$

and

$$-\frac{d^2R}{dt^2} = -\frac{4C}{R^5}\,\frac{dR}{dt} = +\frac{4C^2}{R^9} \tag{18}$$

Since $\frac{d^2R}{dt^2}$ cannot exceed the gravitational acceleration at the surface of the star, from Eq. (18) we obtain a lower limit for R below which no hydrostatic equilibrium is possible

$$R > \left[\frac{4C^2}{MG}\right]^{1/7} = R_\ell \qquad (19)$$

Let $M = 20\ M_\odot$, $\eta = 0.1$, we find the limiting radius R_ℓ to be

$$R_\ell \approx 10^9$$

$$C = 1.08 \times 10^{31} \text{ (in c.g.s. units)}$$

The corresponding temperature as estimated from Eq. (16) is roughly $3\times10^9\ ^oK$. This temperature is below the iron-helium disintegration temperature ($\sim6\times10^9\ ^oK$). However, we remark that the above estimate is very crude. The only conclusion we can draw is that qualitatively both the annihilation neutrino process and the iron-helium disintegration process are capable of causing a massive star to collapse, and a detailed analysis is necessary to tell which process acts first.

(3) Simple Implications of Eq. (6) (The Energy Balance Equation)

From the sign of Eq. (6) we see that the entropy S is a decreasing function of t. For a gas that is not too degenerate, one finds

$$S \propto \ln (T^K/\rho) \qquad (20)$$

where $K = 1.5$ in a non-relativistic gas and $K = 3$ in the relativistic limit.

We can obtain a rough relation between average values of ρ and T for a star from the virial theorem. We have the simple relation

$$\rho \propto T^3 \qquad (21)$$

Eq. (21) holds for a normal star whose composition is homogeneous and whose self-gravitational energy is much less than its rest energy. Substituting Eq. (21) into Eq. (20), we find that as long as the gas is non-relativistic, S is a decreasing function of T, and Eq. (6), (20), and (21) are compatible. If the gas becomes relativistic, Eq. (20) and (21) give a constant S, independent of the temperature and density of the star. This is incompatible with Eq. (6), which states that S must decrease with time. Eq. (6) describes the energy balance relation and must be true. Hence Eq. (21) cannot hold in stars with strong neutrino emission. Since Eq. (21) is obtained with the assumption that the star is homogeneous, the breakdown of it also means that a star homogeneous in composition cannot remain homogeneous in structure in the presence of strong neutrino emission. In some parts of the star the density must rise much faster than the third power of T. This is demonstrated in the numerical example described below.

(4) <u>A Simple Numerical Example of the Development of a Degenerate Core in a Massive Star</u>

Since $\frac{dR}{dt} \propto R^{-9}$, the acceleration term in Eq. (1) may be neglected until the very end of stellar collapse. We now present a model calculation. The pressure term includes the electron and positron pressure, radiation pressure, and ion pressure. All other quantities (pressure, energy, neutrino rate, etc.) are obtained from exact equations describing them

The masses are taken to be 10 $M_\odot$ and 20 $M_\odot$ respectively,

and the initial condition is arbitrarily set by the temperature-density relation

$$\rho \propto T^{1.5} \tag{22}$$

The proportionality constant is determined by the mass. The initial temperature at tne center is taken to be 10^9 ^oK.

It is found that the central density ρ_c has a very steep dependence on the central temperature T_c. We have

$$\rho_c \propto T_c^9 \quad (M = 10\ M_\odot) \tag{23}$$

$$\rho_c \propto T_c^5 \quad (M = 20\ M_\odot) \tag{24}$$

The density and temperature profiels are shown in Figs. 1, 2, 3, and 4. The amazing result is the following: initially we have a reasonable amount of temperature gradient, but it gradually disappears as the the star evolves. There is a very heavy concentration of matter towards the center. In fact, inthe 10 $M_\odot$ model, when T_c (central temperature) is around $4x10^9$ ^oK, the center becomes quite degenerate.

From these two models, we can say that, in general, a degenerate core will develop in the core of a star with strong neutrino dissipation. In certain stars, the degenerate core may induce instability. The mass contained in the degenerate core must not exceed the Chandrasekhar mass limit. Once the mass limit is exceeded, there does not exist any equilibrium, and collapse may take place, independent of iron-helium conversion process.

The fact that the neutrino processes can induce a collapse is illustrated in Figure 5. In Figure 5, the central density is plotted as a function of time. It diverges at some

finite time, meaning a collapse.

These two models are highly simplified, but they indicate that conclusions based on simple qualitative arguments used previously to obtain pre-supernova models are not reliable. A full program to investigate pre-supernova models is underway.

III. Energetics of a Collapsing Star — Neutrinos

(1) Impossibility of Ejecting Enough Matter to Become a White Dwarf

In order that a massive core may eject enough mass to become a white dwarf, some physical mechanism must be present to eject almost all of its mass into space.

It is difficult to imagine a practical mechanism to achieve this. However, we shall now demonstrate on energetic grounds that a star cannot eject enough matter into space for the remaining star to become a white dwarf. The gravitational energy of a star is GM^2/R. Since no cold star of mass $> 1.4\ M_\odot$ may exist (see Introduction), an energy of the amount

$$\frac{G(M^2 - 2M_\odot{}^2)}{R}$$

is needed to dissociate the star from its gravitational binding. This energy must be supplied gravitationally by the remaining core of the star of mass around $M_\odot$, radius r. Hence, to the first approximation

$$\frac{G(M^2 - 2M_\odot{}^2)}{R} \approx \frac{GM_\odot{}^2}{r} \qquad (25)$$

Consider $M \approx 20\ M_\odot$, and a radius of 10^9 cm (which corresponds to a temperature of around 1 mev when instability occurs), we find

$r \sim R/400$. The density of the remaining core will be $(400)^3 = 6.4 \times 10^7$ higher than that of the star before collapse takes place. From Eq. (16), the density at the instant of collapse is estimated to be roughly 10^7 g/cm^3. Hence the remaining core will have a density of at least 10^{13} g/cm^3, which is the density of a neutron star. If we apply results of our numerical model calculation, because there is a higher concentration of mass in the center, even less massive stars may have trouble getting rid of their mass.

This example is cited just to demonstrate that when a massive star collapses, there is no possibility that the remaining core will be a white dwarf. Colgate further demonstrates that shock waves generated during the collapse phase will enable the star to achieve neutron star density at even lower stellar masses. Moreover, the energy released during gravitational collapse will be mostly in the form of neutrinos and cannot be of more than limited use.

(2) Neutrino Losses During Stellar Collapse

The density ρ is related to the Fermi momentum p_F as follows:

$$\rho = \frac{1}{3\pi^2 h^3} p_F^3 = \frac{(m_y c)^3}{3\pi^2 h^3} x^3 \qquad (26)$$

$$x = p_F/m_y c \qquad (27)$$

where m_y is the mass for the particle y. As we have seen, during the evolution of a star, at best $\rho \propto T^3$. For relativistic energy (for electrons) $E_F \approx p_F c$. Hence, at best the degeneracy parameter E_F/kT will be a constant. Otherwise E_F/kT will always increase (gas becomes degenerate).

In the following, we shall assume that the ratio E_F/kT will be roughly 1. With this assumption, if we find the relaxation time for neutrino loss to be short compared with the dynamical time scale, then we can regard the gas as collapsing under zero temperature conditions.

At $T = 6 \times 10^9\ ^oK$ ($kT \sim 0.5$ mev) the most dominant neutrino process is the annihilation process. This is because all other processes involve a threshold energy of the order of the energy difference between neutrons and protons. At higher temperature, the threshold becomes unimportant and all neutrino processes must be considered. Moreover, at low density ($\rho < 10^7$ g/cm^3, say) the mean free path of neutrinos is longer than the physical dimension of the star. At high density, the mean free path of neutrinos can no longer be considered long compared with the physical dimension of the star. The rate of dissipation of neutrinal energy is then less than the production rate.

At $kT \sim m_\mu c^2$, the creation of π meson pairs and μ meson pairs is not negligible. Because the neutrinos associated with the μ meson do not interact with electrons to the lowest order, the dissipation of stellar energy through μ-neutrino production is more important. Table 1 lists the electron, π, μ meson equilibrium densities as a function of temperature.

(3) Neutrino Cross-Sections

When the energy difference between neutron and proton can be neglected, the cross-section for first-order neutrino processes is given by

$$\sigma \frac{v}{c} = CG^2 p_\nu^{\ 2} \qquad (28)$$

where G is the weak interaction constant, v is the relative velocity of the initial system of particles ($\sim$c) and p_ν

is the final neutrino momentum. C is a numerical constant close to unity. Eq. (28) is valid if $p_\nu \lesssim 1$ Bev/c.

(4) Neutrino Losses During Collapse

As the density of the star increases, the mean free path of neutrinos becomes smaller. Once the mean free path of the neutrinos becomes smaller than the dimension of the star, the neutrino energy loss rate is no longer the same as the production rate. Let the radius of the star be R, the mean free path for neutrinos be λ, the total stellar neutrino energy content be U_ν, then the neutrino luminosity L_ν for a star is

$$L_\nu = \frac{U_\nu}{\frac{R}{c}\cdot\frac{R}{\lambda}} \tag{29}$$

In equilibrium, U_ν (the neutrino energy density) is approximately the same as the radiation energy density.

A recent experiment demonstrated that there are two kinds of neutrinos, ν_e and ν_μ. In the lowest order, ν_e does not interact with μ-meson and ν_μ does not interact with electrons. Thus ν_μ can be scattered by an electron.

Since the average energy of ν_μ coming from π-decay and μ-decay is of the order of $m_\mu c^2$ (or less), inverse μ capture reaction cannot occur favorably. Until $kT \approx m_\mu c^2$ ($T \sim 10^{12}$ ^oK), the number density for μ mesons is small. At the temperature ($T = 1.6\times10^{11}$ ^oK) when the loss of energy due to π decay becomes very important, the mean free path for ν_μ-μ scattering is 10^8 cm, which is around the radius of the star ($\sim 7\times10^7$ cm). Hence μ-neutrinos can dissipate stellar energy rapidly while e-neutrinos cannot. Table 2 summarizes our result. The relaxation time for dissipating stellar energy is shortest at roughly

Table 1. Electron, π Meson, and μ Meson Number Densities as a Function of Temperature

T is measured in ^{o}K, n in particles/cm^3.

$\log_{10}T$	$\log_{10}n_e$	$\log_{10}n_\mu$	$\log_{10}n_\pi$
10	31.31		
10.2	31.91		
10.4	32.51		
10.6	33.11	21.7	19.27
10.8	33.71	27.1	26.37
11	34.31	30.5	28.47
11.2	34.91	32.8	31.69
11.4	35.51	34.4	33.69
11.6	36.11	35.6	35.09
11.8	36.71	36.5	36.18
12	37.31	37.2	37.0
12.2	37.91	37.89	
12.4	38.51	38.5	
12.6	39.11	39.1	

$T = 6.3 \times 10^{10}\ ^{o}K$, and again at $T \sim 4 \times 10^{11}\ ^{o}K$. The corresponding relaxation times are roughly 3×10^{-3} and 10^{-4} seconds respectively. This is roughly the time scale for the star to relax to zero temperature configuration discussed previously.

Since this time scale is of the same order of magnitude for light to travel across the star, we conclude the core collapses under zero temperature conditions ($\frac{E_F}{kT} >> 1$).

We have not included in our calculations the red shift of neutrinos due to the gravitational field of the star. The red shift caused by the gravitational field of the star will at most decrease the net loss energy rate by a factor of 2.

IV. Static Structure of Stars at Zero Temperature

This problem is discussed in full detail by Salpeter[8] in a separate paper in this proceeding. We shall emphasize certain points.

As the density of matter approaches 10^{12} g/cm^3, inverse beta processes alter the composition of matter for neutron-rich elements. At $\rho \gtrsim 10^{12}$ g/cm^3, only free neutrons and a small amount of electrons and protons exist. At $\rho \sim 10^{15}$, hyperons begin to come into existence. The structure of a zero temperature star (made of Fermi gas) as a function of the central density ρ_c is shown in Figure 6. We see the mass M first increases as the central density until $\rho_c \sim 10^9$ g/cm^3, then M decreases as ρ_c rises further. When ρ_c approaches 10^{13} g/cm^3, then M increases again. Very soon we come to a second maximum, and M decreases when $\rho_c > 10^{16}$ g/cm^3. It may be emphasized that the second maximum exists even for a perfect Fermi gas.

Since the interparticle spacing in typical neutron stars is of the order of 10^{-13} cm or less (roughly the size of elementary particles) it is not possible to exclude the structure of elementary particles from our discussion. However, some general results can still be obtained without particle physics. First, we examine the limitatıons on the limitations on the equation of state.

(1) The Unimpotance of Quantum-Gravitational Effects

A well-known theorem in Riemannian geometry[9] states that in a given space with a non-Euclidean metric, it is always possible to find a coordinate transformation such that locally at any given point, the metric $g_{\mu\nu}$ may be reduced to that for a Minkowskian space, that is,

$$g'_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (30)$$

Unless the $g_{\mu\nu}$'s change considerably over the dimension of elementary particles ($\sim \frac{\hbar}{mc} \approx 10^{-13}$cm), it is not necessary to consider quantum effects of gravitational fields. Even in the extreme case when the time metric g_{oo} vanishes at the center of a neutron star, quantum effects may still not enter. According to Oppenheimer and Volkoff[2], one can write generally, in the center of a neutron star:

Table 2

NEUTRINO LOSS RATES IN A NEUTRON STAR

TEMPERATURE °K	DENSITY g/cm^3	ENERGY DENSITY ergs/cm^3	NEUTRINO PRODUCTION RATE (ergs/sec cm^3)		MEAN FREE PATH (cm)		RADIUS OF STAR (cm)	RELAXATION TIME sec.	
			ν_e	ν_μ	ν_e	ν_μ		ν_e	ν_μ
10^{10}	10^7	1.6×10^{26}	10^{26}		10^{10}		10^9	1	
	4×10^7	10^{27}	3×10^{27}		1.6×10^9		6.3×10^8	3×10^{-1}	
	1.6×10^8	6.3×10^{27}	2×10^{29}	4×10^{12}	2.5×10^8	> Star	4×10^8	3×10^{-2}	
	6.3×10^8	4×10^{28}	1.3×10^{31}	7×10^{21}	4×10^7	> Star	2.5×10^8	3×10^{-3}	
	2.5×10^9	2.5×10^{29}	10^{33}	9×10^{28}	6.3×10^6	> Star	1.6×10^8	0.1	3
10^{11}	10^{10}	1.6×10^{30}	EQ.	10^{31}	10^6	> Star	10^8	0.3	0.1
	4×10^{10}	10^{31}	EQ.	2×10^{34}	1.6×10^5	> Star	6.3×10^7	0.5	5×10^{-4}
	1.6×10^{11}	6.3×10^{31}	EQ.	2×10^{36}	2.5×10^4	> Star	4×10^7	1	3×10^{-5}
	6.3×10^{11}	4×10^{32}	EQ.	5×10^{37}	4×10^3	> Star	2.5×10^7		10^{-5}
	2.5×10^{12}	2.5×10^{33}	EQ.	EQ.	6×10^2	~ Star	1.6×10^7		10^{-4}
10^{12}	10^{13}	1.6×10^{34}	EQ.	EQ.		~ Star	10^7		10^{-3}

$$g_{oo} = \left(1 - \frac{2m}{Rc}\right)^{\frac{1}{2}} \exp\left[-\frac{1}{c^2}\int_o^{P_c} \frac{dP}{\frac{P}{c^2} + \rho}\right] \qquad (31)$$

when the mass of a neutron star approaches the critical mass, $P_c \to \infty$ and the integral in the exponential diverges logarithmically as one approaches the center, and g_{oo} vanishes as $1/P$. $(P \to \infty)$

Unless $P \to \infty$ abruptly over a dimension of an elementary particle, g_{oo} cannot change substantially. This would not occur since the pressure is itself a macroscopic concept. Hence the center of neutron stars is not likely to be the dwelling place of quantum gravitational field theory.

(2) Relativistic Limits on the Equation of State

General relativity does set up certain limits for the equation of state on the basis of positive (or negative) definiteness of the stress energy tensor, and that signals cannot propagate at a speed faster than light speed. These limitations manifest themselves in the relation of pressure to energy. Since the speed of sound v_s for a given medium is given by

$$v_s = c\sqrt{\frac{dP}{d\epsilon}} \lesssim c \qquad (32)$$

(ϵ is the energy density), integrating Eq. (32) one obtains

$$p \lesssim \epsilon \qquad (33)$$

For a non-interacting as, the trace of the energy momentum tensor

must be positive[10]. This sets up an upper limit for P:

$$P \lesssim \frac{1}{3}\epsilon$$

Recently, Zel'Dovich[11] demonstrated that for a gas composed of classical point charges, interacting through a classical vector field with a finite mass the relativistic limit of Eq. (33) can be achieved.

In any case, the pressure cannot exceed the energy density. This excludes incompressible fluid as a physical equation of state.

(3) Model for Neutron Stars. Limits on the Value of the Central Density and Their Implications

A very elementary theorem tells one that the self-gravitational energy of a body cannot exceed its proper mass or energy, otherwise energy can be "manufactured" by accumulating matter. This sets up a limit on the central density of an object if gravitational binding is required. Applying Eq. (26) to neutron matter, we have

$$\rho \approx 6\times10^{15}\left[\frac{p_F}{m_n c}\right]^3$$

If $p_F >> m_n c$, then the kinetic energy of the neutron gas will exceed the rest energy. Since the maximum self-gravitational energy is Mc^2, this can only mean that when the average density of a neutron star is much greater than 6×10^{15} g/cm^3, there can be no gravitational binding.

Indeed, the binding energies of most neutron star models become positive when one crosses the mass maximum in Figure 6[12]. Only for some pathological equation of state can one get gravitational binding at arbitrarily high density.

It thus seems that in practice it is impossible to accumulate matter until the central density becomes infinite. Hence the singularity $g_{oo} = 0$ (obtained from Eq. (31) by requiring $\rho \sim p \to \infty$) discussed by Wheeler, cannot take place statically by accumulating matter. The inclusion of strange particles may change the critical mass discussed above, but one still cannot build up a singularity $g_{oo} = 0$ just by accumulating masses. This is true in all model neutron stars studied so far.

Unlike the Schwarzschild singularity, the singularity $g_{oo} = 0$ cannot be removed by a mere coordinate transformation. This implication of the singularity $g_{oo} = 0$ is discussed by J. A. Wheeler in a separate paper in this proceeding[13].

When one accumulates masses, one first comes to the so-called second crushing point (the mass maximum near $\rho_c \sim 10^{16}$ g/cm^3 in Figure 6). From then on, one has only a dynamical collapse. The value of the mass maximum, however, depends sensitively on the equation of state. It ranges from 0.3 $M_\odot$ to 3 $M_\odot$.

(4) Schwarzschild Singularity and the Ultimate Mass Limit

For a point mass, the Schwarzschild solution for the metric has tne form

$$ds^2 = g_{ij}dx^{ij} =$$

$$-\frac{1}{1 - \frac{2GM}{rc^2}} - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2 + (1 - \frac{2GM}{rc^2})dt^2 \qquad (34)$$

For a mass m, a singularity occurs at $r_s = \frac{2GM}{c^2}$. That is,

$ds^2 = \infty$, Consequently, no light signal can cross the boundary defined by $r = r_s$*. For massive objects like the sun, the Schwarzschild radius is around 2.6 km, which is about the size of a neutron star of the same mass with a density of around 10^{16} grams.

This singularity has been the subject of discussion among relativists ever since the solution was found. Is it a real singularity? Or is it just a singularity related to a particular coordinate system?

By a change of coordinates of $r \to \hat{r}$, $\theta \to \theta$, $\varphi \to \varphi$, $t \to t$, where $\hat{r}$ is related to r by the following**

$$r = \left(1 + \frac{GM}{2\hat{r}c^2}\right)^2 \hat{r} \tag{35}$$

Eq. (34) becomes

$$ds^2 = -\left(1 + \frac{m'}{2\hat{r}}\right)^4 (d\hat{r}^2 + \hat{r}^2 d\theta^2 + \hat{r}^2 \sin^2\theta d\varphi^2) + \frac{(1 - m'/2\hat{r})^2}{(1 + m'/2\hat{r})^2} dt^2 \tag{36}$$

where

$$m' = GM/c^2$$

is the geometrized mass. In Eq. (36) the singularity in $\hat{r}$ coordinates disappears. Hence it seems that the singularity at $r = r_s$ is a coordinate singularity.

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*We might remark, such behavior was first predicted by Laplace on the basis of a corpuscular theory of light.

**This coordinate system is known as the isotropic coordinate system.

Robertson[14] has demonstrated that by a coordinate transformation one can eliminate this singularity in the following sense: An observer in flat space observes the free fall of a test particle towards the singularity. The time it takes the test particle to cross the boundary (which is at $r_s = 2GM/c^2$) is infinite; this is because $g_{oo} = 0$ at the boundary, and the test particle suffers infinite time dilatation as it approaches the singularity. However, if the observer moves with the test particle, the time duration that he observes for the test particle (and himself) to cross the boundary is <u>finite</u>. Hence, in the opinion of many relativists, this singularity is a <u>coordinate</u> singularity since it can be removed by a proper choice of coordinate systems (transformation to a co-moving frame).

We regard the Schwarzschild singularity as a <u>physical singularity</u> in the following sense. Given a matter distribution such that initially nowhere within the matter distribution is $r_s \leq \frac{2GM}{c^2}$. An observer observes this distribution of matter in a nearly flat space (since almost all points of the universe are locally nearly flat). Since it takes an infinite time for this distribution of matter to cross the Schwarzschild singularity, we may regard the Schwarzschild singularity as a real, physical singularity. Once matter is found in a non-singular state, within the finite life time of the universe it will remain in the non-singular state. Similarly, if a Schwarzschild singularity

were found in nature, it was created with the universe.

Since all massive stars undergo supernova explosion, their core will inevitably collapse to extreme high densities. It might be worthwhile to investigate whether they collapse asymptotically to become Schwarzschild singularities or not.

(5) Dynamical Collapse

To date, there is only one analysis of dynamical, relativistic collapse. This is the famous investigation of Oppenheimer and Snyder[15]. Although their analysis involves some unlikely physical assumptions (zero pressure), it will serve as a good example. In the absence of pressure, the collapse may be treated analytically. Oppenheimer and Snyder computed the world trajectories of material bodies falling towards a common center under the gravitational field produced by these material bodies. They obtained the following result: To an external observer, these particles fall toward the Schwarzschild singularity asymptotically, and the total time of fall is infinite. This is because of the infinite time dilatation effect mentioned earlier. To a local observer, the time is finite, and for the case of the sun, it is of the order of one day. Since the star continues to fall indefinitely, the star is always in some dynamical motion, and this is consistent with the fact that no static structure for such star exists if its mass exceeds a certain limit.

(6) Angular Momentum and Stellar Collapse

A rapidly rotating star need not preserve cylindrical symmetry, as the classical analysis of Jacobi showed. Modern versions of Jacobi's discussion have been extensively inveéstigated by Chandrasekhar[16].

Angular momentum conservation sets a limit on stellar collapse. The angular momentum $I\omega$ is a conserved quantity (ω is the angular velocity). For a normal star, $R \approx 10^{11}$ cm and the period of rotation is one day. Then

$$I\omega \propto R^2\omega \approx 6\times10^{17} \tag{37}$$

The centrifugal force f_c per unit mass at the equator of the star is

$$f_c = \frac{v_{eq}}{R} \tag{38}$$

where $v_{eq} = R\omega$ is the speed at the equator. Since $f_c < MG/R^2$ (otherwise the star will be torn apart by rotation), we have the upper limit on R:

$$\frac{(R^2\omega)^2}{R^3} < \frac{MG}{R^2}$$

or:

$$R > \frac{(R^2\omega)^2}{MG} \tag{39}$$

This is tne minimum radius a rotating star may collapse to, and this is the radius at which bifurcation of a spherical star to non-symmetrical shape takes place. Using $M = 20\ M_\odot$, and the value of $R^2\omega$ given by Eq. (37), we find

$$R > 10^8 \text{ cm} \tag{40}$$

and at this limiting value of R the angular velocity ω is around 1 rad/sec, and the period is around 1/10 sec. It is highly dubious that such rapidly rotating stars can still preserve cylindrical symmetry. As an estimate, we apply to our rotating star the formula of gravitational wave radiating power computed for a spinning rod. The radiating power is:

$$P = \frac{32GI^2\omega^6}{5c^5} \tag{41}$$

The rotational energy is $\frac{1}{2} I\omega^2$. Hence the relaxation time τ for the star to lose its angular momentum is:

$$\tau = \frac{5c^5}{64GI\omega^4} \tag{42}$$

For the parameters we consider, we find $\tau \sim 50$ seconds. Hence one need not consider angular momentum as a hindrance to stellar collapse.

V. Observable Features of Neutron Stars

From our discussions, it is seen that during stellar collapse the density of a star will reach that of a neutron star. There is no assurance, however, as to what kind of object one may expect to find after the collapse. Indefinite

collapse state as suggested by Oppenheimer and Snyder can certainly be one solution; on the other hand, stars in certain mass ranges can collapse to form neutron stars while ejecting mass. It seems that the critical mass for a neutron star is quite sensitive to the equation of state. In certain models, the critical mass even exceeds the Chandrasekhar mass limit. In such cases, it is conceivable that neutron stars exist in nature.

It has been suggested previously[3] that the surface temperature of a neutron star is around 10^7 ^oK, and most radiation will be in the X-ray band. It is not possible to locate them by optical means. We now discuss the surface structure of neutron stars in more detail.

The internal structure of neutron stars has been widely studied. The general results (independent of the detailed form of any covariant equation of state) are: The mass is between 0.2 $M_\odot$ to around 1.3 $M_\odot$; neutron stars with mass outside the above range cannot exist. The radius is around 10^6cm. Hyperons as well as neutrons may exist if the density is greater than 10^{15} g/cm^3. Since neutron matter is unstable against beta decay when the density is below 10^{12} g/cm^3, the outer shell of a neutron star must be composed of ordinary matter. Most likely, elements are in an equilibrium state; at $\rho < 10^7$ g/cm^3, Fe^{56} is mostly abundant. However, other elements may exist, and their presence may give clues to the history of formation of neutron stars.

We now estimate the energy content, the internal temperature, and the surface properties of neutron stars.

(1) Internal Energy

There are no energy sources inside a neutron star: the radiated energy comes from the remaining thermal energy of the degenerate neutron gas. The heat capacity C_v is

$$C_v = \frac{\pi^2 k^2}{m_n c^2} \frac{(x_n^2 + 1)^{\frac{1}{2}}}{x_n^2} T = 0.75 \times 10^{-4} \frac{(x_n^2 + 1)^{\frac{1}{2}}}{x_n^2} T \qquad (43)$$

where k is the Boltzmann constant, m_α the mass for particle α, T the temperature, c the speed of light. $x_\alpha = p_F^{(\alpha)}/m_\alpha c$, and $p_F^{(\alpha)}$ is the Fermi momentum for particle α. In gravitationally bound neutron star models, x_n has a value in between 0.1 and 0.5. Taking a value 0.3 for x_n, the total energy E_t of a star is estimated to be

$$E_t = 3.8 \times 10^{47} T_9^2 M_{33} \text{ ergs} \qquad (44)$$

where $T_n = T/10^n$, $M_{33} = M/10^{33}$.

(2) Neutrino Production and Internal Temperature

The plasma neutrino process is the most important neutrino process[17]. From the equilibrium electron density in a typical neutron star, ($M = 10^{33}$ g/cm^3, radius $R = 10^6$ cm, mean density $\rho = 10^{14}$ g/cm^3) an average neutrino emission rate can be estimated; the results are listed in Table 3. The lifetime against neutrino emission decreases strongly with increasing temperature. A maximum in the lifetime exists at $T \sim 10^9$ ^oK is around 10^3 years. This we take to be the typical time scale.

The mean energy of plasma neutrinos at $T \sim 10^9$ ^oK in the

neutron core is around 10 mev. Because of strong degeneracy, the mean free path of 10 mev neutrino in neutron core is fairly long ($\gtrsim 10^4$ cm). One does not need to consider corrections to the neutrino rate due to scattering.

(3) Surface Composition and Structure[18]

As the density drops drastically from 10^{15} g/cm^3 in the center to 10^6 g/cm^3 on the surface, the temperature in these regions remains virtually unchanged. This is because of the high thermal conductivity of degenerate electrons. Hence we can take the temperature to be virtually constant up to $\rho \sim 10^6$ g/cm^3. A small amount of matter supports all temperature gradients.

Near the surface of a typical neutron star, the relativity parameter, $GM/Rc^2 \approx 0.1$. (R is the radius of the neutron star, G is the gravitational constant). As a good approximation, non-relativistic theory may be used. Neglecting the mass contained in the shell, the structure of the outer layer of the atmosphere is given by:

$$\frac{dP}{dr} = - \rho \frac{GM}{r^2} \tag{45}$$

The physical meaning of Eq. (3) is that the gravitational force is balanced by the hydrostatic pressure gradient. The pressure P for a degenerate electron gas is related to ρ through the following equations[5]:

$$P = \frac{1}{24\pi^2} \frac{m_e^4 c^5}{h^3} f(x) \tag{46}$$

$$\rho = \frac{1}{3\pi^2} \frac{m_e^3 c^2}{h^3} x^3 \mu_e m_p \tag{47}$$

$$f(x) = x(2x^2 - 3)(x^2 + 1)^{\frac{1}{2}} + 3 \sinh^{-1} x \tag{48}$$

where $x = p_F^{(e)}/m_e c$; $\mu_e = \langle A \rangle / \langle Z \rangle$, where $\langle A \rangle$ and $\langle Z \rangle$ are the average mass number and atomic number for the outer layer ($\mu_e \sim 2$). Solving Eq. (45), one obtains

$$\rho = 4\times10^{13} \left(\frac{M_{33}}{R_6}\right)^3 \left(\frac{1}{\xi} - 1\right)^{3/2} \left[\left(\frac{1}{\xi} - 1\right) + 7.4\times10^{-3} \frac{R_6}{M_{33}}\right] \tag{49}$$

where $\xi = r/R$, $R_6 = R/10^6$ cm. For $R_6 = M_{33} = 1$, and $\rho = 10^{12}$ g/cm^3, we find $1 - \xi \approx 0.1$. The mass contained in the outer layer (non-neutron matter) amounts only to about 10^{-3} of the star.

Matter is non-degenerate at low density; hence Eq. (49) will break down eventually. In the non-degenerate layer, two physical processes contribute to the opacity of matter to radiation: Compton scattering and photo-ionization of K-shell electrons of Fe^{56}. The cross-section of Compton scattering at $T = 10^9$ ^oK is still quite close to its non-relativistic value, which is a constant $= \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = 6.7\times10^{-25}$ cm^2. The photo-ionization process has a ν^{-3} dependence (ν is the frequency of the photon). At $T > 5\times10^7$ ^oK, there are very few bound electrons, and only Compton scattering need be considered.

We may define the beginning of the non-degenerate layer

by using one of the following three criteria: (i) when the perfect gas pressure ($P = \frac{k}{m_p}\frac{1}{\mu_e}\rho T$) is equal to that for a degenerate gas (Eq. (46)), this gives $\rho \sim 10^6$ g/cm^3 at $T = 10^9$ ^oK. (ii) When the thermal conductivity of electron gas is the same for that due to Compton scattering. This gives $\rho \approx 5\times10^5$ g/cm^3. These two criteria give the same value of ρ to within a factor of 2.

The equation of radiative transfer is

$$\frac{a}{3}\frac{dT^4}{dr} = -\frac{\kappa L}{4\pi c r^2}\rho \tag{50}$$

where L is the total energy flux of the star, κ the so-called Roseland mean of opacity, $\kappa = 0.2$ cm^2/g for Compton scattering [$(\kappa\rho)^{-1}$ is roughly the mean free path of photons], a is the Stefan-Boltzmann constant. Combining Eq. (49) and (45), and using the perfect gas pressure for P, one obtains

$$\frac{3}{a}\frac{k}{\mu_e m_p}\frac{d(\rho T)}{d(T^4)} = \frac{4\pi cGM}{\kappa L} \tag{51}$$

The solutions for Eqs. (50) and (51) are

$$\rho = \frac{4\pi cGM}{\kappa L}\frac{a\mu_e m_p}{3k}T^3 = \frac{1.92\times10^{-18}}{(L/L_\odot)(\kappa/0.2)}M_{33}T^3 \tag{52}$$

and

$$T = \frac{m_p\mu_e}{3k}GM\left(\frac{1}{r} - \frac{1}{R}\right) = 5.4\times10^{11}\left(\frac{1}{\xi} - 1\right)M_{33} \tag{53}$$

where $L_\odot$ = solar energy output rate = 4×10^{33} ergs/sec.

We notice that the thickness of the envelope is

independent of L so long as κ is a constant (or nearly so). For photo-ionization process (bound-free transition)

$$\kappa \sim \frac{\rho}{T^{3.5}} \tag{54}$$

Again, if $\rho \sim T^3$ (Eq. (52)), κ cannot vary too much. Thus, Eqs. (52) and (53) are approximately true throughout the star. One can fit ρ and T from Eq. (52) to those at the boundary of the degenerate layer (the boundary has been discussed before) using L as the fitting parameter. The density of the boundary at $T = 10^9\ ^oK$ is $10^6\ g/cm^3$. Hence we find $L \approx 2 \times 10^3 L_\odot$. This value of L may be further checked by noticing that

$$L = \frac{ac}{4} T^4 4\pi R^2 / n_s = 7 \times 10^{44} \frac{T_9^4}{n_s} \text{ ergs/sec} \tag{55}$$

where n_s is the total number of scatterings an average photon suffers before leaving the star, and T is the internal temperature of the star. The proof of Eq. (54) may be found in reference (3). n_s is given by

$$n_s = \int \kappa \rho dr \tag{56}$$

The integration terminates at roughly the surface temperature of the star. Using Eqs. (52), (53), and a more precise value of κ from a Los Alamos calculation (known as the Los Alamos Code[19], and considered to be the best opacity calculation available today), one finds $n_s = 1.8 \times 10^8$, of which around

10% is contributed by photo-ionization process. From Eq. (54) we find $L = 10^3\ L_\odot$, which is only a factor of 2 different from the value we obtained from fitting boundary conditions. Further, one can integrate Eqs. (50) and (51) numerically throughout the star and find a central temperature corresponding to a certain surface temperature. The integrated temperature at $\rho \sim 10^6$ g/cm^3 at a surface temperature of 10^7 ^oK is 0.8×10^9 ^oK, and at $\rho \sim 10^{12}$ g/cm^3, it is 1.2×10^9 ^oK. Both values are close to the value of 10^9 ^oK that we assumed.

Table 3 lists the photon luminosity as a function of the central temperature, surface temperature, and $\tau \equiv T/(dT/dt)$ (lifetime). Our method of computation will not yield reliable results when $T < 5 \times 10^8$ ^oK, because then photo-ionization process will contribute considerably to the opacity. However, when the internal temperature drops to below 10^8 ^oK, the spectrum of surface emission shifts to the ultraviolet region, for which interstellar absorption is large and the chance for observation is small.

TABLE 3. NEUTRON STAR EMISSION CHARACTERISTICS [20][21]

Internal Temperature ^{o}K	Total Thermal Energy(ergs)	Photon Luminosity ergs/sec	Surface Temperature ^{o}K	Lifetime (photon) years	Lifetime (neutrino) years	Wave length of Maximum in Spectrum (Å)
$5x10^{9}$	10^{48}	$5x10^{37}$	$1.7x10^{7}$	$7x10^{3}$	1	1.9
$2x10^{9}$	$1.6x10^{48}$	$2x10^{37}$	$1.3x10^{7}$	$3x10^{3}$	10^{2}	2.5
10^{9}	$4x10^{47}$	$8x10^{36}$	$1.1x10^{7}$	$1.7x10^{3}$	10^{3}	3
$5x10^{8}$	10^{47}	$2.8x10^{36}$	$0.8x10^{7}$	10^{3}	$>>10^{3}$	4.1

Model Mass = 10^{33} g; Radius = 10^{6} cm.

(4) Spectrum of Surface Emission of Neutron Stars

Although the energy output of a neutron star is high, only a small fraction of it ($\sim 10^{-3}$) will be able to penetrate through our atmosphere[22], and most of the emission will be in the X-ray region. The mean surface density is around 0.1 g/cm^3. With a value of κ = 2.6 at T = 10^7 ^oK (photo-ionization process) most of the emission will come from a layer of thickness around 5 cm ($\sim$ the mean free path of photons). The spectrum will show absorption lines or discontinuities at the ionization energy of K or L electrons; the width of these lines and discontinuities is of the order of kT. From the location of more than one of these discontinuities (or lines) one can determine the red shift and the composition.

The red shift is of the order of $\frac{GM}{Rc^2}$ ($\sim$0.1) and is quite large compared with the Doppler shift caused by the motion of heavenly bodies ($\frac{v}{c} \sim 10^{-4}$). The red shift is proportional to M/R. On the other hand, if we can obtain a good, reliable equation of state from elementary particle theory, we can obtain a theoretical M-R relation. Combining this result with the red shift measurement, the mass of neutron stars can be measured. The surface temperature and the measured flux on the earth, combining with an information on R, will give us a distance, which can be checked against known distances of supernova remnants, if

any. Presently, the measurable flux is around 10^{-8} ergs/sec-cm^2. This means that neutron stars at a few thousand light years can now be detected. In the future, when space observatories are available, it is not beyond the present technology to detect all neutron stars (age $\approx$ 1000 years) in our galaxy. Since at 10Å, interstellar absorption is negligible, one can even detect those behind dust clouds. Supernova explosions obscured by dust clouds may now be observable in the X-ray band, since during the first year of the formation of neutron stars, they have an X-ray luminosity equivalent to 10^4 $L_\odot$.

Moreover, because the lifetime of neutron stars against optical emission is short, one can even observe annual declines of the energy flux and the decrease in temperature. One can get great sensitivity by observing the tail of Planck distribution (e.g., observe 10 kev photons for 1 kev surface temperature).

The rate of occurence of supernovae is around one per 50 → 300 years. Hence the maximum number of experimentally observed neutron stars X-ray sources anticipated cannot exceed, say, 50. At present, only three discrete sources have been resolved.[23][24]

Further examination of these X-ray sources may tell us whether they are neutron stars, and give answers to a very important question.

VI. Conclusion

It is inevitable that massive stars will collapse towards the end of their evolution. During the collapse stage, most of the gravitational energy released will be rapidly converted into neutrinos and dissipated from the core. A density $> 10^{14}$ g/cm^3 will be reached at the center of massive stars during collapse.

It is not known if the remnants of such collapse will cut itself off gravitationally or not. If the remnants do not cut themselves off, they will be observable in the 10Å X-ray band for around one thousand years. The present technology allows us to observe all neutron stars ($\lesssim$ 1000 years old) in our galaxy.

Since the galaxy is transparent in the 10Å X-ray band, "X-ray pictures" of the galaxy may be taken with appropriate apparatus, revealing those supernovae neutron star remnants which would otherwise be obscured by dust clouds.

Although there are indications from recent rocket observations that discrete X-ray sources exist which might be interpreted as neutron stars,[23][24] only future experiments will tell us if these X-ray sources are indeed neutron stars or not.

VII. Acknowledgement

I would like to thank Drs. A.G. W. Cameron, C.W. Misner, E.E. Salpeter for discussions and Miss Jane Keston for help in numerical integrations.

1. For a complete bibliography, see "Introduction to Stellar Structure", S. Chandrasekhar, Chapter XI (Dover, 1951).

2. J. R. Oppenheimer and G. Volkoff, Phys. Rev. 55, 374 (1939)

3. H. Y. Chiu, "Supernovae, neutrinos, and neutron stars" To be published in Annals of Physics. This article contains a somewhat detailed discussion of materials not covered or only briefly mentioned here.

4. F. Hoyle and W. A. Fowler, Astrophys. J. 123, 565 (1960).

5. H. Y. Chiu, Phys. Rev. 123, 1040 (1961).

6. Although we also use the simple relation $\rho \propto T^3$ here, our result only serves as an example and indications for directions of a more detailed calculation program whose preliminary result is also discussed here.

7. Since the amount of nuclear energy left at $T \gtrsim 10^9\ ^oK$ is small, nuclear fuel will be consumed before the temperature of the star is so high that nuclear energy becomes an important term in Eq. (6)

8. E. E. Salpeter, "Superdense Equilibrium Stars", in this proceeding. A review and bibliography may be found in Ref. (3)

9. This is easily proven. The line element ds may be written as

$$ds^2 = \sum_{i,j} g_{ij} dx^i dx^j \qquad \text{(i)}$$

where $\{g_{ij}\}$ is the metric tensor and $\{x^i\}$ is a general set of coordinates. Since $\{g_{ij}\}$ is a symmetric tensor, at a particular point $\{g_{ij}\}$ can be diagonalized by "rotating" the $\{x^i\}$ coordinate system at this point. An additional scale change in the rotated $\{x^i\}$ system reduces the line element to

$$ds^2 = \sum_{i,j} \pm\delta_{ij} dx^i\ dx^j \qquad \text{(ii)}$$

where δ_{ij} is the Kronecker delta and the sign ($\pm$) is chosen so as to be consistent with the signature of Eq. (ii). Since g_{ij} varies with position, obviously this transformation is valid only at the particular point chosen.

10. See, for example, L. Landau and E. Lifshitz, "A Classical Theory of Fields", p. 89 (Translated from the Russian by M. Hamermesh, Addison Wesley, Cambridge, 1961).

11. Y. B. Zel'Dovich, Zhur. Eksp. i Teoret. Fiz. 41, 1609 (1961). Translation: Soviet Phys. JETP 14, 1143 (1962).

12. A. G. W. Cameron, private communication.

13. J. A. Wheeler, paper presented at Conference on Relativistic Astrophysics and Gravitational Collapse, Dallas, Texas, December, 1963.

14. M. P. Robertson, Phil. Mag. 5, 835 (1928).

15. J. R. Oppenheimer and H. Snyder, Phys. Rev. 56, 455 (1939).

16. A series of six articles by S. Chandrasekhar and his collaborators appears in the Astrophysical Journal, vol. 136.

17. B. Adams, M. Ruderman, and C. H. Woo, Phys. Rev. 129, 1383 (1963).

18. The surface of neutron stars resembles that of white dwarfs. See Ref. (1).

19. A. Cox, "Stellar Envelope Coefficients and Opacities", Los Alamos Laboratory pre-print (unpublished).

20. When this paper was finished, the author learned that Dr. D. Morton (to be published) obtained results similar to those given in Table I. However, Dr. Morton did not obtain the neutrino life time.

21. R. Stabler (Cornell thesis, 1960, unpublished) has first considered the observable features of neutron stars. However, he used a simplified form of bound-free opacity (Kramer's Law) all throughout the calculation, and he obtained a surface temperature ~ 3 times lower than that obtained in this paper. Kramer's Law is not applicable to the surface of neutron stars.

22. E. E. Salpeter, "Superdense Equilibrium Stars", reported in the Dallas Conference on Gravitational Collapse, held December 16 - 18, 1963. Proceedings to be published. Visible part of the Planck spectrum at $T \sim 10^7$ ^oK is around 10^{-8} of the total energy output, but a large fraction of ultraviolet light ($\sim 10^{-3}$) can be converted into visible light by interstellar gas surrounding the neutron star.

23. H. Gursky, R. Giacconi, F. R. Paolini, and B. Rossi, Phys. Rev. Letters 115, 30 (1963).

24. H. Friedman, Paper presented in Astronomical Society Meeting December, 1963 (Washington, D. C.) (To be published).

Figure Captions

Figure 1.

The temperature T of a 10⊙ star is plotted as a function of the shell mass M_r (M_r is the mass contained inside a sphere of symmetry of radius r) for several time sequences (marked 1 through 8 in order of increasing time). The flattening of the temperature gradient is apparent.

Figure 2.

The density ρ as a function of M_r for the same star as in Fig. 1 and for the same time sequences. The development of a dense degenerate core is obvious.

Figure 3.

The temperature T of a 20⊙ star is plotted as a function of M_r for eight time sequences (marked 1 through 8). Note there is very little flattening of the temperature gradient as compared with that for a 10⊙ star.

Figure 4.

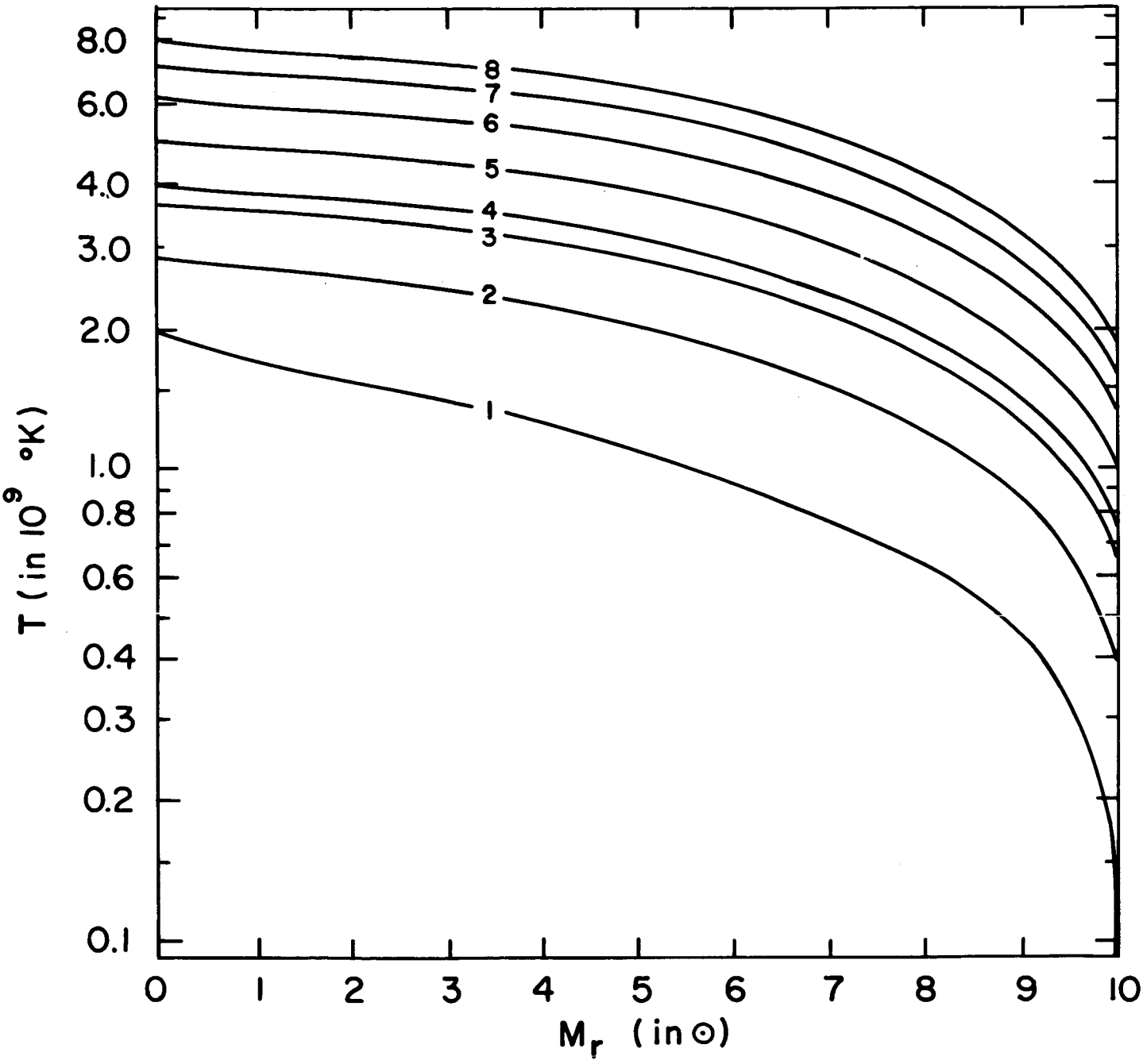
The density ρ as a function of M_r for the same star as in Fig. 3 and the same time sequences. The development of a dense core is obvious.

Figure 5.

The central temperature T_c of the 10M⊙ star (described in Fig. 1 and 2) is plotted as a function of time. T_c diverges at some finite time.

Figure 6.

Mass versus central density curve for a cold degenerate star. Dotted curve CH is that by Chandrasekhar, that marked O-V is due to Oppenheimer and Volkoff. The solid curve is due to Wheeler. Chandrasekhar used the non-relativistic approximation for the stellar structure equation and a perfect Fermi gas for the equation of state. Oppenheimer and Volkoff used the same equation of state but a relativistic form for the stellar structural equations. Wheeler took into account the beta-transition.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T (in 10^9 °K)
8
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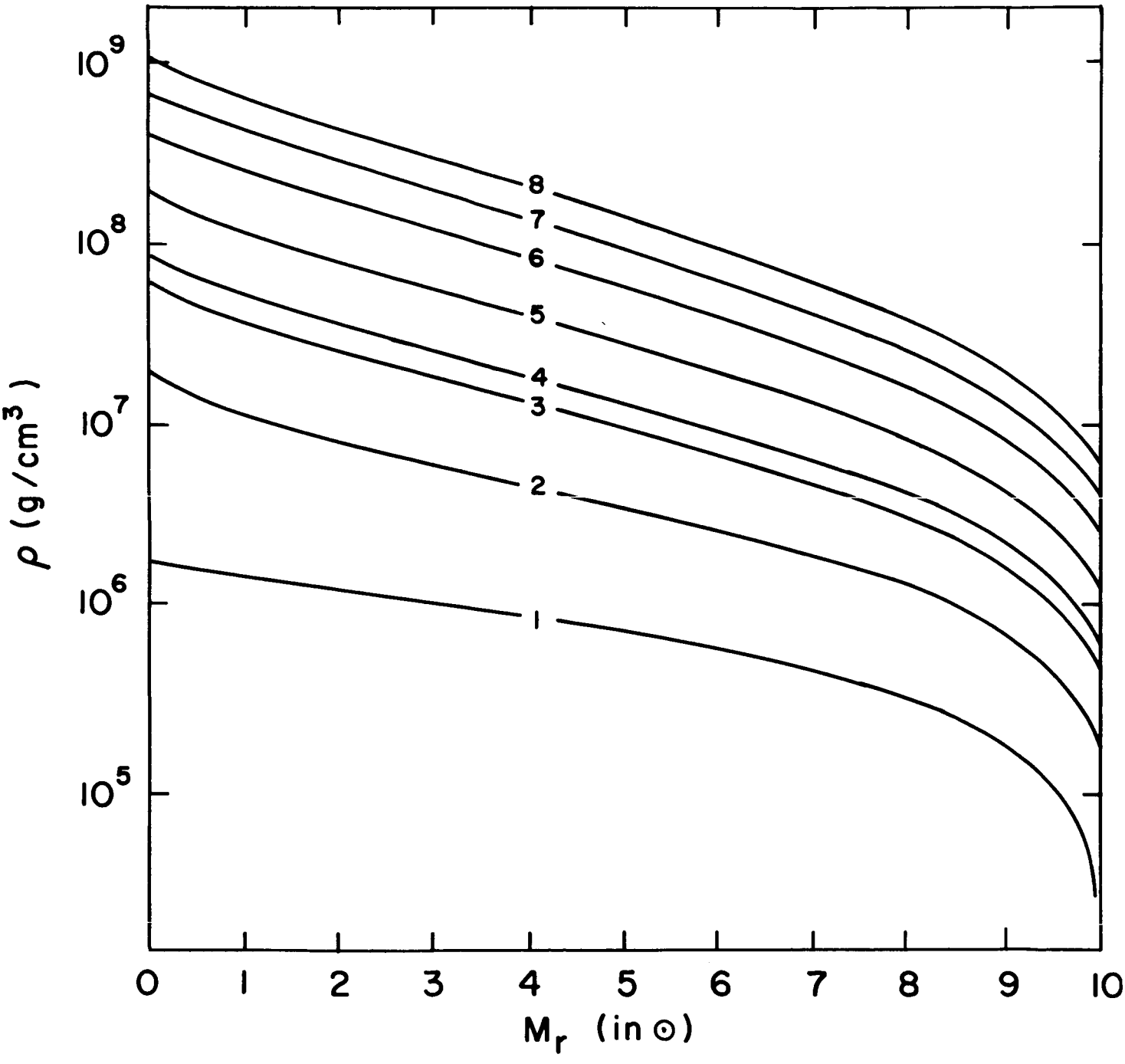
10
M_r (in ⊙)

Fig. 1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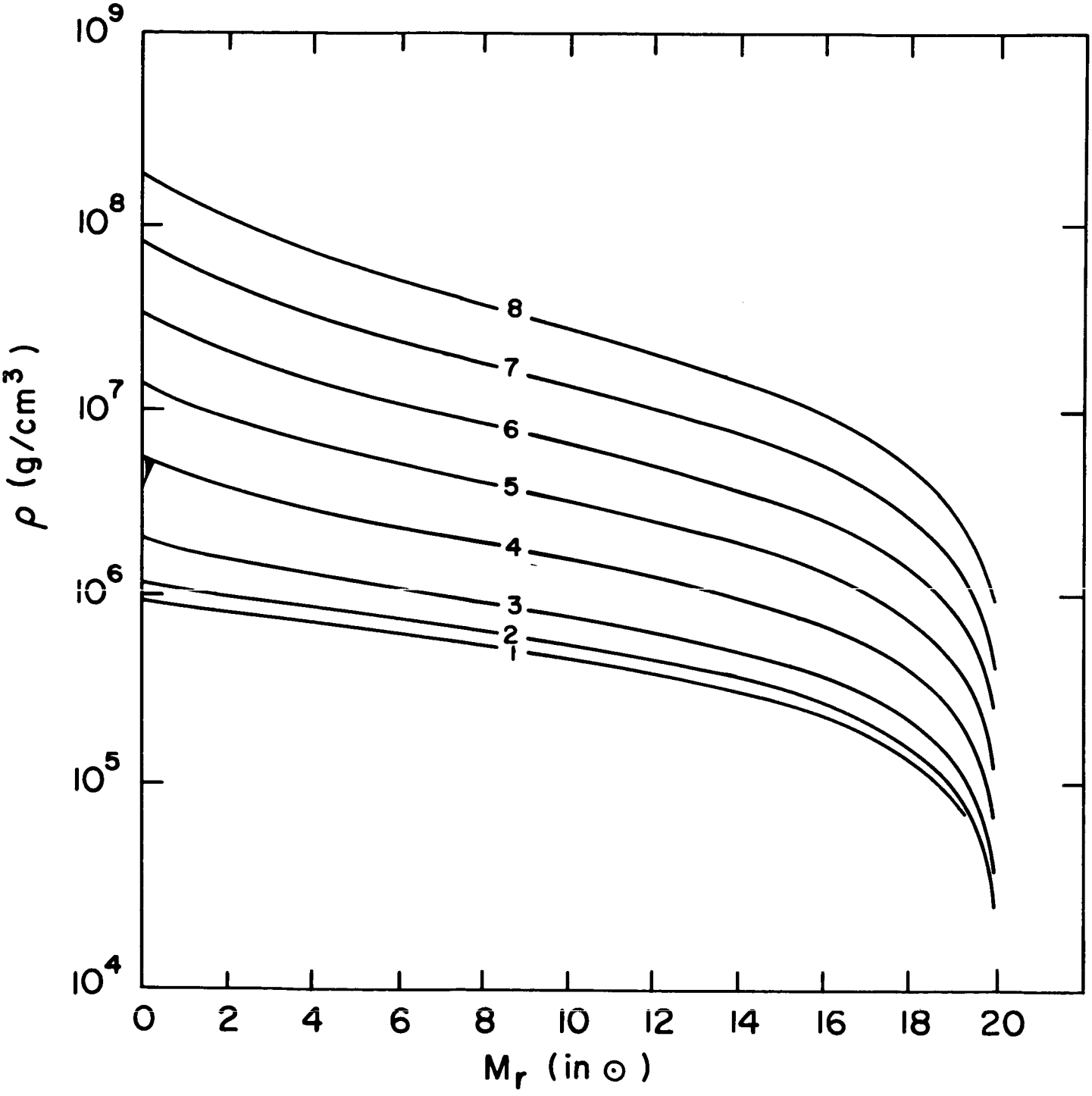
10
M_r (in $\odot$)

Fig. 2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10^4
ρ (g/cm^3)
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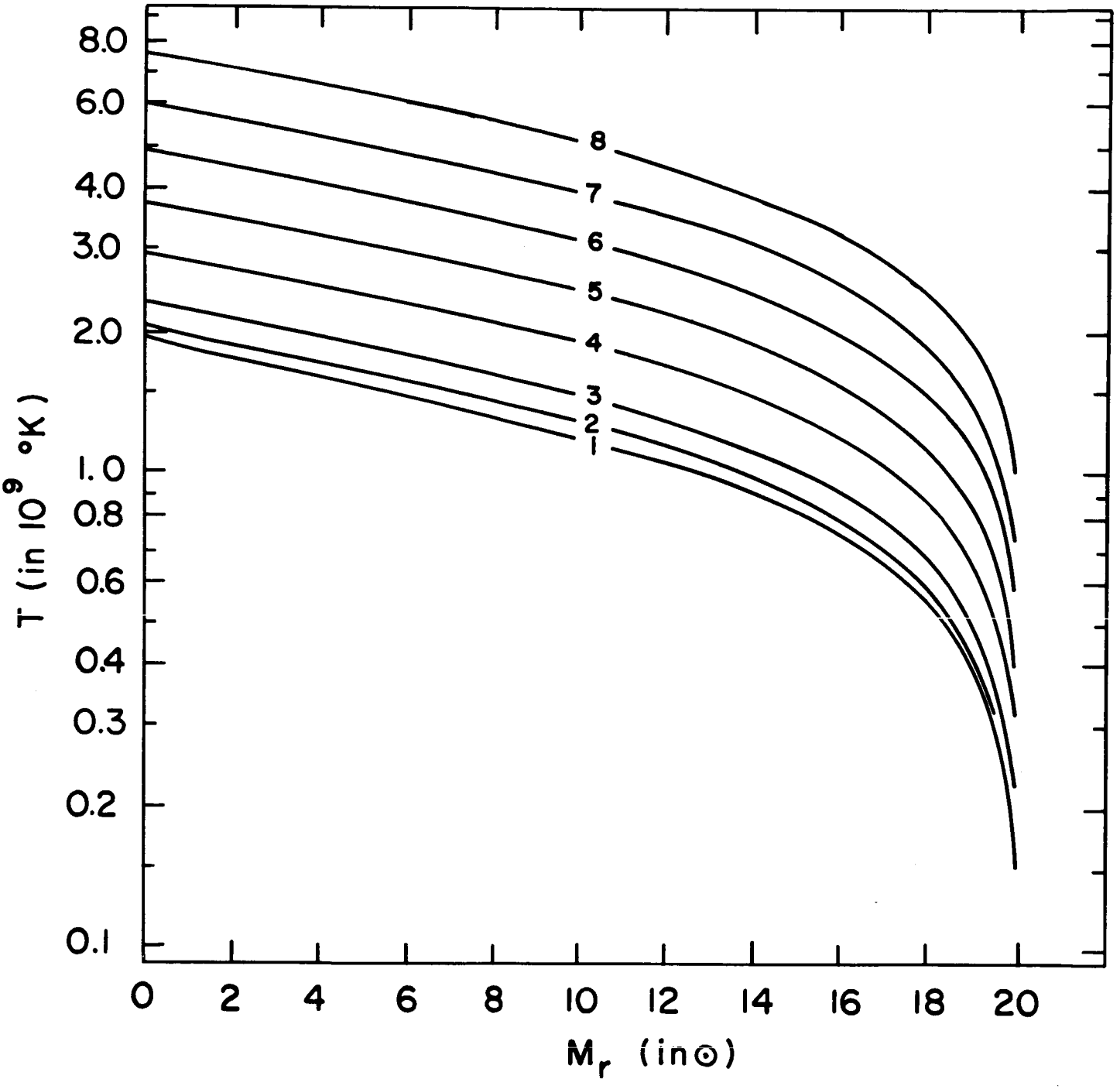
M_r (in $\odot$)

Fig. 3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20
M_r (in ⊙)

Fig. 4

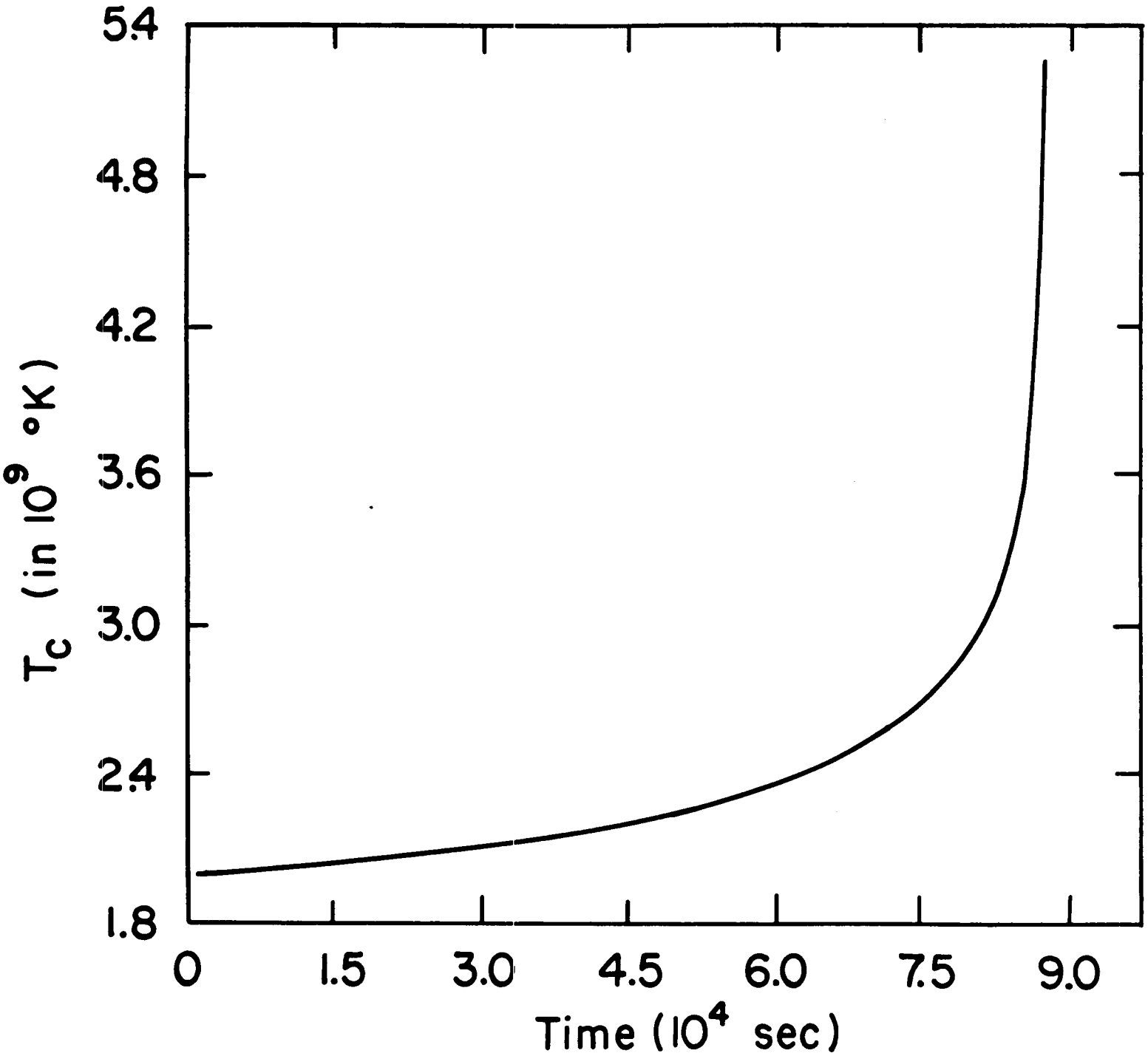

Fig. 5

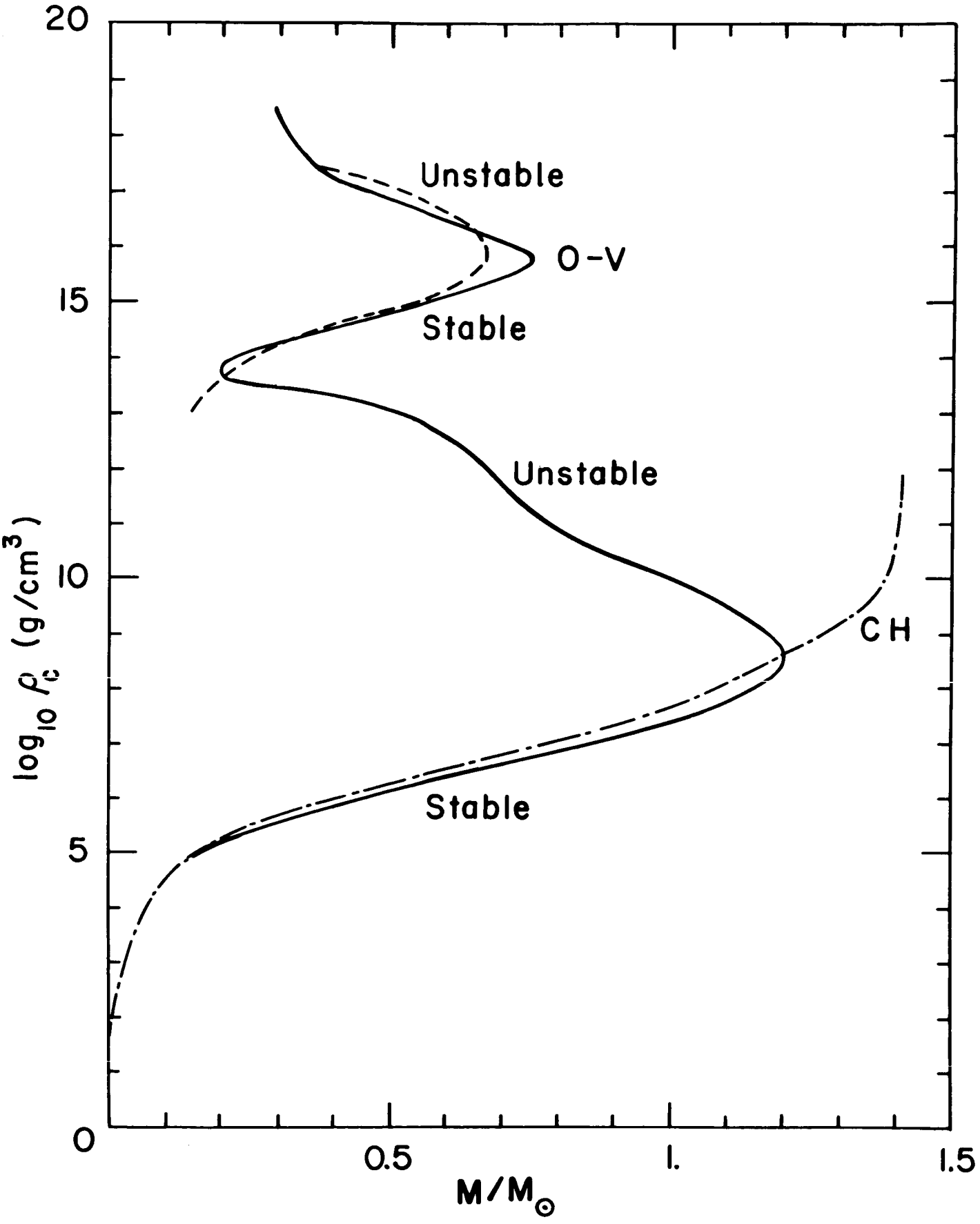
20
15
10
5
0
Unstable
O-V
Stable
Unstable
C H
Stable
$\log_{10} \rho_c$ (g/cm^3)
0.5
1.
1.5
M/M$_\odot$

Fig. 6